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First Semester MCA Degree Examination, June/July 2018 Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Define ?
 - i) proposition
 - ii) logical equivalence.

Show that the compound propositions : $[p \land (\neg q \lor r)]$ and $[p \lor (q \land \neg r)]$ are not logically equivalent. (06 Marks)

Prove that for any propositions p, q, r the compound proposition.

 $[(p \lor q) \land \{(p \to r) \land (q \to r)\}] \to r$, is a tautology.

(05 Marks)

c. Give a direct proof of the statement "The square of an odd integer".

(05 Marks)

OR

- 2 a. For any propositions p, q, r verify the distributive laws:
 - i) $p \land (q \land r) \Leftrightarrow (p \lor q) \land (p \lor r)$

ii)
$$p \land (q \lor r) \Leftrightarrow (p \land q) \lor (p \land r)$$
.

(06 Marks)

b. Without constructing the truth table simplify the following logical statement.

$$(p \vee q) \wedge [\sim \{(\sim p) \wedge q\}].$$

(05 Marks)

- c. Negate and simplify each of the followings
 - i) $\exists x, [p(x) \lor q(x)]$
 - ii) $\forall x, [p(x) \land \neg q(x)]$
 - iii) $\forall x, [p(x) \rightarrow q(x)]$
 - iv) $[\exists x [p(x) \lor q(x)] \rightarrow r(x).$

(05 Marks)

Module-2

- a. Define a power set. Prove that for a finite set S, has n-elements and its power set has 2ⁿ-elements. (06 Marks)
 - b. Define Relative Compliment (difference of two sets) of two sets A and B. For any three non-empty sets A, B, C prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$. (05 Marks)
 - c. Let R be a relation on the set A = {1, 2, 3, 4} defined by xRy is 'x divides y'. Prove that (A, R) is a Poset. Draw its Hasse diagram. (05 Marks)

OR

- a. In a sample of 100 logic chips, 23 have a defect D₁, 26 have defect D₂, 30 have a defect D₃, 7 have defect D₁ and D₂, 8 have defects D₁ and D₃, 10 have defects D₂ and D₃ and 3 have all the three defects. Find the number of chips having i) at least one defect ii) no defect (05 Marks)
 - b. Define one-one and onto functions. Let $f: z \rightarrow z$ be defined by $f(a) = a + 1 \ \forall \ a \in z$, find whether f is one-one, onto or both. (05 Marks)
 - c. Let $A = \{1, 2, 3, 4\}$

 $R = \{(1, 1), (1, 2), (2, 3), (2, 4), (3, 4), (4, 1), (4, 2)\}$

 $S = \{(3, 1), (4, 4), (2, 3), (2, 4), (1, 1), (1, 4)\}$

Write he relation matrices of: i) M_R ii) M_S iii) M(RoS) iv) M(SoR). (06 Mark)

Module-3

- 5 a. State Pigeon-hole principle. ABC is an equilateral triangle whose sides are of length lcm, if we select 5-points inside the triangle, prove that at least two of these points are such that the distance between them is less than ½cm.
 - b. Find the number of permutations of the letters of the word INSTITUTION:
 - i) How many begin with I
 - ii) How many of these begin with I and end with N?
 - iii) In how many words are the three T's are together?

(06 Marks

c. A bank pays 6% on savings compounding the interest annually. If a person deposits R 1000/- on the first day of the May. How much will this deposit be worth after a year?

(05 Marks)

OR

- 6 a. Among the integers 1 to 200, determine the number of digits that are:
 - i) Divisible by 2 or 5 or 9
 - ii) Not divisible by 2 or 5 or 9
 - iii) Not divisible by 5.

(06 Marks)

b. Find the coefficient of x^4y^5 in the expansion of $(2x - 3y)^9$.

(05 Marks)

c. The sequence $\{a_n\}$ – is defined by recursively $a_1 = 4$, $a_n = a_{n-1} + n$ for $n \ge 2$. Find $\{a_n\}$ in explicit form.

<u> Module-4</u>

- 7 a. Define:
 - i) Mutually exclusive events
 - ii) Conditional probability
 - iii) Mutually independent events.

(05 Marks)

- b. A problem is given to four students A, B, C and D whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ respectively. Find the probability that the problem is solved. (06 Marks)
- c. For any three events A, B, C prove that:

 $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C).$ (05 Marks)

- a. If A and B are events with $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{4}$, find P(A|B) P(B|A), $P(\overline{A}|\overline{B})$, $P(\overline{B}|\overline{A})$. (06 Marks)
 - Define event and sample space, hence prove that $P(A) + P(\overline{A}) = 1$. A is any event.

(05 Marks)

A husband and wife appear for an interview for two vacancies in the same post. The probability of husband's selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$. what is the probability that i) Both of them will be selected ii) only one of them selected iii) none of them selected. (05 Marks)

Module-5

- Define the terms with one example for each:
 - i) Complete graph
 - ii) matrix representation of graphs
 - iii) incidence matrix of graph.

(06 Marks)

Find the number of vertices, the number of edges and the degree of each vertex in the following undirected graph in Fig.Q9(b). (05 Marks)

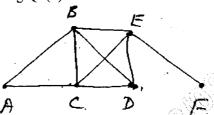


Fig.Q9(b)

Define Bipartite graph, with an example. How many edges the graph K₃₆ contain. (05 Marks)

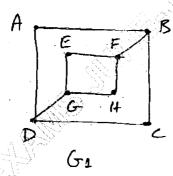
OR

Define subgraph and spanning subgraph. 10

(06 Marks)

Determine whether the graphs shown in Fig. Q10(b) are isomorphic.

(05 Marks)



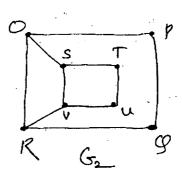


Fig.Q10(b)

Explain Konigsberg bridge problem.

(05 Marks)

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